Torque Calculations for Vertically Oriented Power Screws

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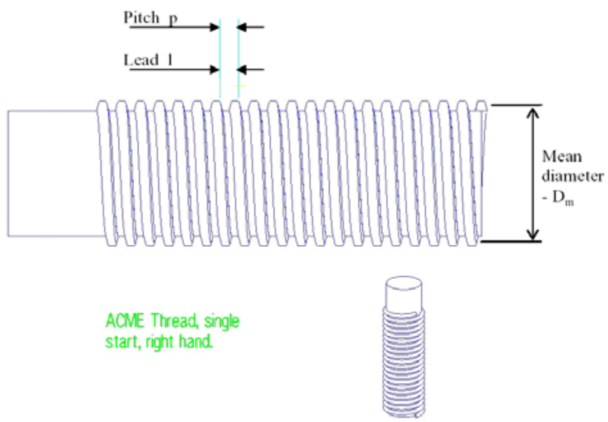
Abstract. This is a brief exercise in computing the lifting power of a 3/8 in power screw coupled with a stepper motor rated at 45 in ozf.

Basic Equations

This is the Scientific Article - with Instructions shell document which provides a sample layout of a scientific article. Replace the text in this shell with your own.



Single start square thread power screw.

Note that the pitch (p) and lead (l) are equal. This can be tricky. In US units threads are reported using the units threads/inch, so the pitch for our unwrapped thread can be calculated:

$$p_{us} = l_{us} = \frac{1}{threads\,\mathrm{in}^{-1}}$$

In SI they are measured in terms of millimeters/thread. Be careful with this.

$$p_{si} = l_{si} = \mathrm{mm}\,thread^{-1}$$

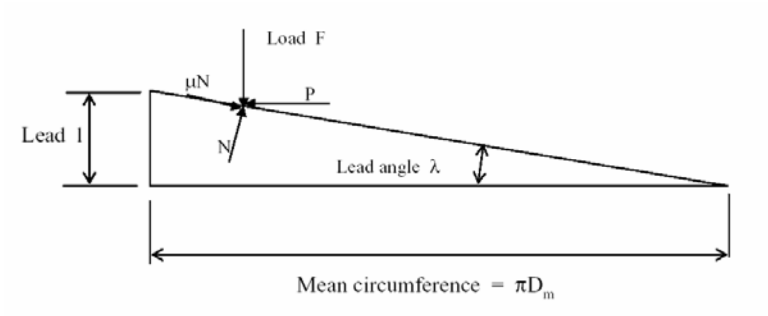
With the pitch we can calculate the mean diameter of the power screw using the expression:

$$D_m = D_{\max} - \frac{p}{2}$$

American screw thread charts typically report a maximum and minimum diameter for threaded shafts so a simple mean can be taken between the two.

$$D_m = \frac{D_{\max}+D_{\min}}{2}$$

If we unwrap one turn of a square thread,



we get the ramp shown above. This has a slope given by the expression:

$$\tan(\lambda) = \frac{p}{D_m}$$

Now consider the load F being pushed up the ramp by force P. From force equilibrium we get:

$$P - N\sin(\lambda) - \mu N\cos(\lambda) = 0$$
$$N\cos(\lambda) - F - \mu\sin(\lambda) = 0$$

Where:

N	-	force vector from friction
μ	-	friction coefficient (0< μ <1)

From these equations we can develop the expression for the force required to raise or lower the load:.

$$P_{raise} = \frac{F(\sin\lambda - \mu\cos\lambda)}{\cos\lambda - \mu\sin\lambda}$$

$$P_{lower} = \frac{F(\mu\cos\lambda - \sin\lambda)}{\cos\lambda + \mu\sin\lambda}$$

We can then develop an expression for torque by using the relationship:

$$T = \frac{PD_m}{2}$$

If we do not use a rolling bearing on the screw we will have a substantial axial load and must calculate the friction torque and add it to the lifting or lowering torque.

$$T_c = \frac{F\mu D_c}{2}$$

Where D_c, the mean diameter of the load-bearing collar is calculated:

$$D_c = \frac{D_o+D_i}{2}$$

Example (US)

We have a 3/8”-24 threaded rod. In this case we can get the mean diameter of the rod by taking the average of the major and minor diameters off of the thread chart

http://www.engineersedge.com/screw_threads_chart.htm

instead of trying to calculate it from the pitch.

$D_{m(us)} = \frac{0.375+0.3239}{2} = 0.349\,45\,\text{in}$

Let us also set our friction coefficient to:

$\mu = 0.15$

and our pitch and lead to

$p = l = \frac{1}{24} = 0.04\,166\,666\,667\,\text{in}$

Using the slope expression

$\tan(\lambda) = \frac{p}{\pi D_m}$

$\tan(\lambda) = \frac{4.166\,666\,667\times10^{-2}\,\text{in}}{\pi(0.349\,45\,\text{in})}$

$\lambda = 0.03793\,546\,744\,\text{rad}$

Setting the load to a unit of 1 lb, the force to raise that pound is calculated from the expression:

$P_{raise} = \frac{F(\sin\lambda - \mu\cos\lambda)}{\cos\lambda - \mu\sin\lambda}$

Inserting values we get:

$P_{raise} = \frac{(1)(\sin(3.793\,546\,744\times10^{-2}) + (0.15)\cos(3.793\,546\,744\times10^{-2}))}{\cos(3.793\,546\,744\times10^{-2}) - (0.15)\sin(3.793\,546\,744\times10^{-2})}$

$P_{raise} = 0.189\,029\,832\,1\,\text{lbf}$

From which we can calculate the lifting torque required using the expression:

$T_{raise} = \frac{P_{raise} D_m}{2}$

And inserting values

$T_{raise} = \frac{(0.189\,029\,832\,1\,\text{lbf})(0.349\,45\,\text{in})}{2} =$

$T_{raise} = 0.03302\,823\,741\,\text{in lbf}$

Now calculating the torque requirement from the axial load imposed by the load-bearing collar we first calculate the mean collar diameter:

$$D_c = \frac{D_o + D_i}{2}$$

$$D_c = \frac{0.5 + 0.34945}{2} = 0.424725 \text{ in}$$

And then the torque:

$$T_c = \frac{F\mu D_c}{2}$$

$$T_c = \frac{(1)(0.15)(0.424725)}{2} = 0.031854375 \text{ in lbf}$$

For a total torque requirement of:

$$T_{total} = T_{raise} + T_c$$

$$T_{total} = 0.03302823741 + 0.031854375 = 0.06488261241 \text{ in lbf}$$

Converting to more convenient units (in ozf) noting that there are 16 ozf in 1 lbf.

$$T_{total} = 0.06488261241(16) = 1.038121799 \text{ in ozf}$$

Now supposing we have a stepper motor that is rated at 45 in ozf. Using this power screw shaft it can lift

$$\frac{45 \text{ in ozf}}{1.038121799 \text{ in ozf lbf}^{-1}} = 43.34751476 \text{ lbf}$$

The efficiency of this power screw can be calculated using the expression:
Inserting our calculated values we find:

$$\eta = \frac{Fp}{2\pi T_{total}}$$

$$\eta = \frac{(1)(0.04166666667)}{2\pi(1.038121799)} = 0.006387936338$$

Or 0.63%